AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

A-level
FURTHER MATHEMATICS
Paper 3 Mechanics

Thursday 13 June 2019 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

1 A spring has natural length 0.4 metres and modulus of elasticity 55 N

Calculate the elastic potential energy stored in the spring when the extension of the spring is 0.08 metres.

Circle your answer.

[1 mark]

0.176 J   0.44 J   0.88 J   1.76 J

$k = \frac{\lambda}{L} = \frac{55}{0.4} = 137.5$

$PE = \frac{1}{2}kx^2 = \frac{1}{2} \times 137.5 \times 0.08^2$

$= 0.44 \text{ J}$

2 A particle has an angular speed of 72 revolutions per minute.

Find the angular speed in radians per second.

Circle your answer.

[1 mark]

$\frac{6\pi}{5}$ $\frac{12\pi}{5}$ 12π 24π

$72 \text{ rpm} = \frac{72}{60} = \frac{6}{5}$ revs per second

One revolution $= 2\pi$ radians

$\therefore$ angular speed $= \frac{6}{5} \times 2\pi = \frac{12\pi}{5} \text{ rad s}^{-1}$

3 A disc, of mass m and radius r, rotates about an axis through its centre, perpendicular to the plane face of the disc.

The angular speed of the disc is ω.

A possible model for the kinetic energy E of the disc is

$$E = km^a r^b \omega^c$$

where a, b and c are constants and k is a dimensionless constant.

Find the values of a, b and c.

[3 marks]

$[E] = M L^2 T^{-2}$

and $E = k m^a r^b \omega^c$

$\therefore M L^2 T^{-2} = [m]^a [r]^b [\omega]^c$
$= M^a L^b T^{-c}$

$a = 1$

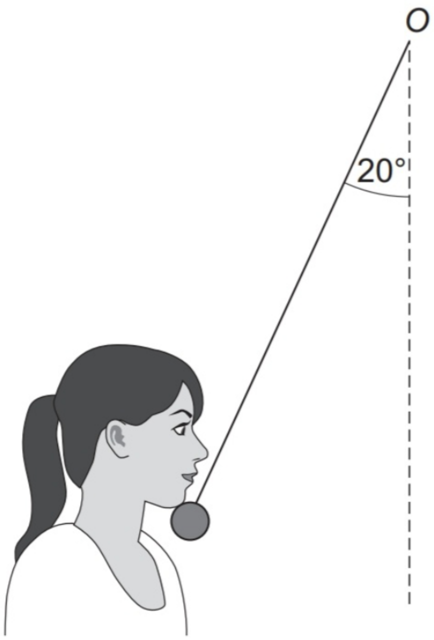
$b = 2$

$c = 2$

4 **In this question use $g = 10\,\text{m}\,\text{s}^{-2}$**

An inelastic string has length 1.2 metres.
One end of the string is attached to a fixed point O.
A sphere, of mass 500 grams, is attached to the other end of the string.

The sphere is held, with the string taut and at an angle of 20° to the vertical, touching the chin of a student, as shown in the diagram below.



The sphere is released from rest.

Assume that the student stays perfectly still once the sphere has been released.

4 (a) Calculate the maximum speed of the sphere.

[3 marks]

GPE at start $= 0.5 \times 10 \times (1.2 - 1.2\cos 20)$
$= 0.362$

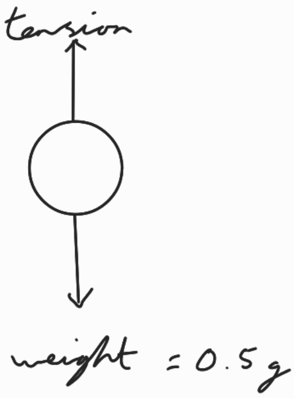
KE at vertical $= \frac{1}{2} \times 0.5 \times v^2$
$= \frac{v^2}{4}$

at max speed, all GPE is converted to KE
$\therefore \frac{v^2}{4} = 0.362$

$v = \sqrt{1.447} = 1.203$
$= 1\,\text{ms}^{-1}$ (1sf)

4 (b) Find the maximum tension in the string.

[3 marks]



centripedal acceleration $= \frac{v^2}{r}$

$\therefore$ By Newton II : $T - 0.5 \times 10 = 0.5 \times \frac{1.203^2}{1.2}$

$T = 5.60$

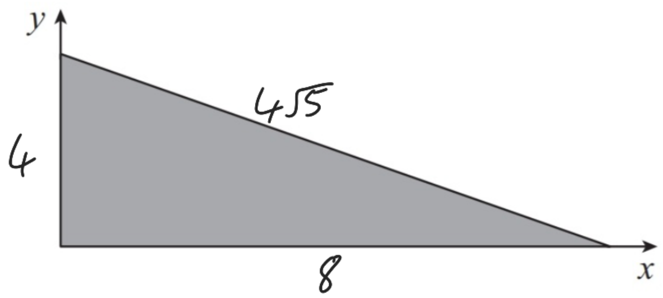
$= 6\text{N}$ (1sf)

4 (c) State, with a reason, whether or not the sphere touches the student's chin again after it has been released.

[2 marks]

No, as there is air resistance that will slow the ball down on its journey, preventing it from touching the student.

5 The triangular region shown below is rotated through 360° around the x-axis, to form a solid cone.



The coordinates of the vertices of the triangle are (0, 0), (8, 0) and (0, 4).

All units are in centimetres.

5 (a) State an assumption that you should make about the cone in order to find the position of its centre of mass.

[1 mark]

Assume the cone is uniform.

5 (b) Using integration, prove that the centre of mass of the cone is 2 cm from its plane face.

[5 marks]

Equation of slanted line:

$$y = \left(\frac{4-0}{0-8}\right)x + 4$$

$$y = 4 - \frac{1}{2}x$$

Mass of cone = volume × density

$$\text{Mass} = \frac{1}{3} \times \pi \times 4^2 \times 8 \times \rho$$

$$= \frac{128\pi\rho}{3}$$

Let $\bar{x}$ = the distance between plane face and the C.O.M.

$$\therefore \frac{128\cancel{\pi\rho}}{3} \times \bar{x} = \cancel{\pi\rho}\int_0^8 x\left(4 - \frac{1}{2}x\right)^2 dx$$

$$\frac{128\bar{x}}{3} = \int_0^8 \left(16x + \frac{1}{4}x^3 - 4x^2\right) dx$$

$$\frac{128\bar{x}}{3} = \left[8x^2 + \frac{x^4}{16} - \frac{4}{3}x^3\right]_0^8$$

$$\frac{128\bar{x}}{3} = \left(8 \times 8^2 + \frac{8^4}{16} - \frac{4}{3} \times 8^3\right) - 0$$

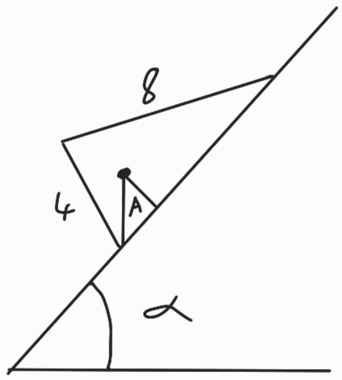
$$\frac{128\bar{x}}{3} = \frac{256}{3}$$

$$\bar{x} = 2$$

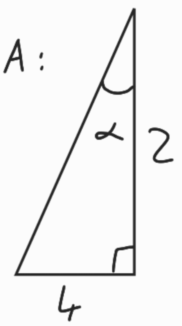
5 (c) The cone is placed with its plane face on a rough board. One end of the board is lifted so that the angle between the board and the horizontal is gradually increased. Eventually the cone topples without sliding.

5 (c) (i) Find the angle between the board and the horizontal when the cone topples, giving your answer to the nearest degree.

[2 marks]



triangle A:



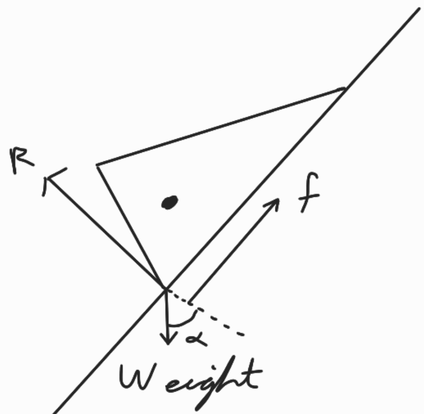
$\tan \alpha = \frac{4}{2}$

$\alpha = 63°$

5 (c) (ii) Find the range of possible values for the coefficient of friction between the cone and the board.

[3 marks]

On the point of sliding at this angle:



$F = \mu R$

$mg \sin \alpha = \mu mg \cos \alpha$

$\frac{mg \sin \alpha}{mg \cos \alpha} = \mu$

$\therefore \mu = \tan \alpha = 2$

$\Rightarrow \mu > 2$

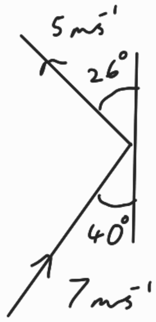
6 A ball moving on a smooth horizontal surface collides with a fixed vertical wall. Before the collision, the ball moves with speed $7\,\text{m s}^{-1}$ and at an angle of $40°$ to the wall.

After the collision, the ball moves with speed $5\,\text{m s}^{-1}$ and at an angle of $26°$ to the wall.

Model the ball as a particle.

6 (a) Find the coefficient of restitution between the ball and the wall, giving your answer correct to two significant figures.

[3 marks]



Newton's law of restitution (applied perpendicular to wall)

$7\sin 40 \times e = 5\sin 26$

$e = \frac{v}{u} = \frac{5\sin 26}{7\sin 40} = 0.49$

6 (b) Determine whether or not the wall is smooth.

Fully justify your answer.

[3 marks]

If the wall is smooth, speed parallel to the wall will stay constant through the collision.

$u = 7\cos 40 = 5.36$

$v = 5\cos 26 = 4.49$

$5.36 \neq 4.49$

$\therefore$ wall cannot be smooth

(As the components of the velocity parallel to the wall are not equal)

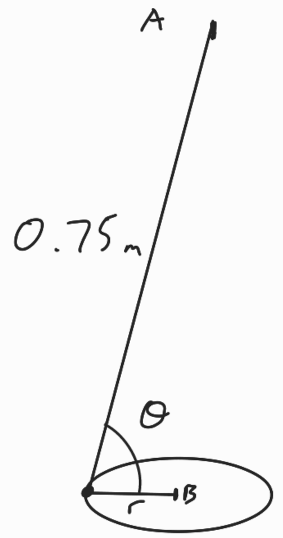
7 A particle of mass 2.5 kilograms is attached to one end of a light, inextensible string of length 75 cm. The other end of this string is attached to a point A.

The particle is also attached to one end of an elastic string of natural length 30 cm and modulus of elasticity λ N. The other end of this string is attached to a point B, which is 60 cm vertically below A.

The particle is set in motion so that it describes a horizontal circle with centre B. The angular speed of the particle is 8 rad s^{-1}

Find λ, giving your answer in terms of g.

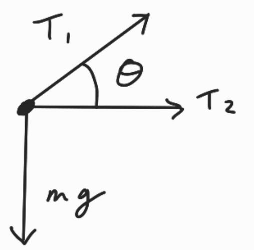
[9 marks]



$\sin\theta = \frac{60}{75} = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\therefore\ r = 0.75\cos\theta = 0.45$



Resolve vertically

$T_1 \sin\theta = 2.5g$

$T_1 = \frac{25g}{8}$

For circular motion, horizontal components of tension must equal the centripedal force:

$T_2 + T_1\cos\theta = mrw^2$

$T_2 + \frac{25g}{8} \times \frac{3}{5} = 2.5 \times 0.45 \times 8^2$

$T_2 = 72 - \frac{15g}{8}$

$$\text{Tension} = \frac{\lambda x}{l}$$

$$\therefore\ 72 - \frac{15g}{8} = \frac{\lambda}{0.3} \times (0.45 - 0.3)$$

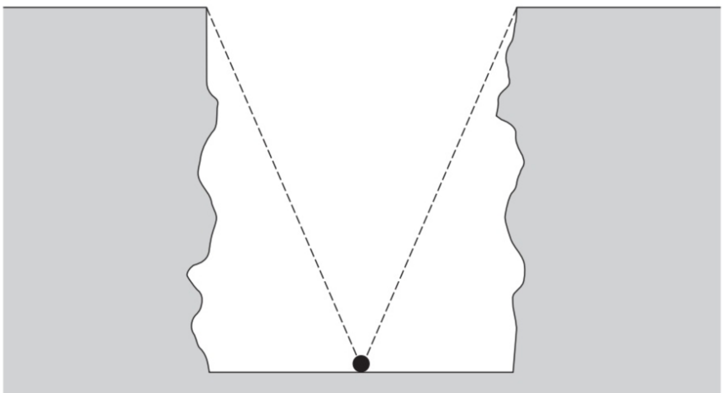
$$\Rightarrow \lambda = 144 - \frac{15g}{4}$$

8 **In this question use $g = 9.8\,\text{m s}^{-2}$**

A 'reverse' bungee jump consists of two identical elastic ropes. One end of each elastic rope is attached to either side of the top of a gorge.

The other ends are both attached to Hannah, who has mass 84 kg

Hannah is modelled as a particle, as shown in the diagram below.



The depth of the gorge is 50 metres and the width of the gorge is 40 metres.

Each elastic rope has natural length 30 metres and modulus of elasticity 3150 N

Hannah is released from rest at the centre of the bottom of the gorge.

8 (a) Show that the speed of Hannah when the ropes become slack is $30\,\text{m s}^{-1}$ correct to two significant figures.

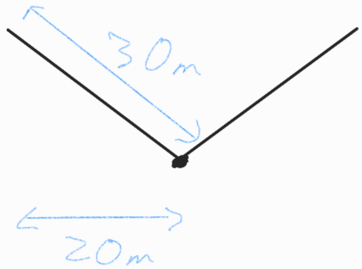
[6 marks]

Initially: rope length $= \sqrt{50^2 + 20^2} = 10\sqrt{29}$

extension $= 10\sqrt{29} - 30 = 23.9$ m

$$EPE = 2 \times \frac{1}{2} \times \frac{3150}{30} \times 23.85^2 = 59735\text{ J}$$

When unstretched :



GPE gained $= 84 \times 9.8 \times (50 - \sqrt{30^2 - 20^2})$

$= 84 \times 9.8 \times 27.639$

$= 22753 \text{ J}$

$\therefore$ the rest of the initial EPE must have been converted to KE.

$59735 - 22753 = \frac{1}{2} \times 84 v^2$

$36982 = 42 v^2$

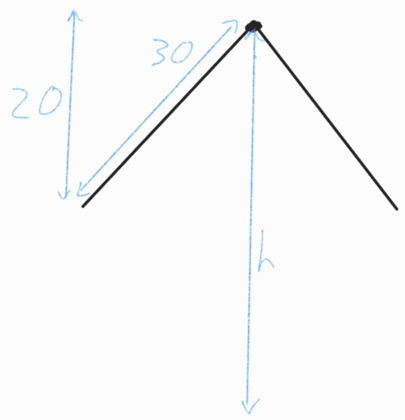
$v = \sqrt{880}$

$v = 29.6\ldots$

$= 30 \text{ ms}^{-1}$ (2sf)

8 (b) Determine whether Hannah is moving up or down when the ropes become taut again.

[5 marks]



String is taut at height h:

$$h = 50 + \sqrt{30^2 - 20^2} = 72.4\,m$$

Maximum GPE = 59735 J

$$\therefore 59735 = 84 \times 9.8\, h_{max}$$

$$h_{max} = 72.6\, m$$

Strings become taut just below maximum height, at which point Hannah is moving upward.